Maximizing Efficiency of Wireless Power Transfer with Resonant Inductive Coupling

Henry Liu
#0277-________
October 12, 2011
Subject: Physics
Supervisor: B. Stephenson

Sir Winston Churchill Secondary School
International Baccalaureate Program

3534 words
Abstract

Wireless power transfer has been a dream sought after ever since the discovery of electricity. Beginning with electromagnetic induction, physicists and engineers have attempted to increase the efficiency with which electrical power can be transferred through the air. Recently, a team of scientists at MIT found that by adding a capacitor in parallel with the coil used to transmit the energy, the efficiency of the wireless power transfer could be increased dramatically. Two resonant circuits, one transmitting, one receiving, could be configured such that both have the same resonant frequency, and the ringing of one circuit with a sinusoid would cause the other to “pick up” the oscillations and begin ringing as well. This is called resonant inductive coupling.

In this experiment, I investigated the relationship between the resonant frequency used by the system and the efficiency of the power transfer between the transmitting and receiving circuits. Two identical resonant circuits were constructed and the frequencies of both varied by changing the capacitors used. I measured efficiency by deriving the power input and power output using measurements from a digital multimeter and an oscilloscope. A function generator tuned to the resonant frequency that was being tested supplied the sinusoidal waveform. This experiment investigated the 20 – 600 kilohertz range of resonant frequencies.

Results showed that as resonant frequency increases, efficiency increases proportionally, though with a slower and slower rate as frequencies got higher and higher. I also established that there is a maximum limit for efficiency, which is achieved by an infinitely high frequency. However, I also showed that by a certain frequency, it is pointless to attempt to increase frequency even more for higher efficiency, as the increase in efficiency is minimal and insignificant.

284 words
Acknowledgements

I would like to thank several teachers at Sir Winston Churchill Secondary School for their support and guidance throughout the course of this investigation. First and foremost, I express my thanks to my extended essay supervisor, Mr. Stephenson, for providing assistance and guidance during many of what were the most stressful weeks I have ever endured, and for taking the time to read and examine what I imagine must seem like an unending torrent of sentences upon sentences. I also express my thanks to the members on the Eng-Tip forums, who have aided me greatly in the extremely technical details of my apparatus and setup, and provided tips and pointers on common practices and procedures in the engineering community. Finally, I am extremely grateful to my electronics teacher, Mr. Gibbens, who provided many helpful words of advice and numerous critical suggestions, and from whom I borrowed most of the materials and instruments for this investigation.
# Table of Contents

Abstract .................................................................................................................................................. 1  
Acknowledgments ............................................................................................................................... 2  

1. Introduction ..................................................................................................................................... 4  
   1.1 Background ............................................................................................................................... 4  
   1.2 Objective .................................................................................................................................... 4  
2. Experimental Design ....................................................................................................................... 5  
   2.1 Measuring Efficiency ............................................................................................................... 5  
   2.2 Apparatus Overview .............................................................................................................. 6  
   2.3 Materials List .......................................................................................................................... 6  
   2.4 Descriptions of Certain Materials .......................................................................................... 7  
   2.5 Procedure .............................................................................................................................. 8  
3. Data Analysis .................................................................................................................................. 10  
   3.1 Processed Raw Data .............................................................................................................. 10  
   3.2 Measuring correlation and significance .................................................................................. 12  
   3.3 Determination of Relationship .............................................................................................. 14  
4. Discussion ...................................................................................................................................... 16  
   4.1 Data and Results ..................................................................................................................... 16  
   4.2 Sources of Error and Improvements ...................................................................................... 17  
5. Conclusion ...................................................................................................................................... 19  
Appendix A .......................................................................................................................................... 20  
Works Cited .......................................................................................................................................... 21  
Works Consulted ................................................................................................................................... 21
1. Introduction

1.1 Background

Ever since the advent of electrical devices, the necessity for power cords has prompted research into the wireless transmission of electrical energy. One method of doing so involves two coils tuned to the same resonant frequency. The principal idea is to make a primary LC circuit oscillate at its resonant frequency, which, similar to how one tuning fork of the same frequency as another will ring if the other is hit, causes a secondary LC circuit with the same resonant frequency to “pick up” the radiated energy (Figure 1-1). This is called resonant inductive coupling. It is a form of electrodynamic induction.

![Figure 1-1: Schematic diagram of resonant inductive coupling](image)

1.2 Objective

Because a parallel LC circuit draws, ideally, no current at its resonant frequency, in theory all power drawn from the AC source will be emitted as EM waves to be received by the secondary. Naturally, then, resonant inductive coupling shows great promise in maximizing the efficiency of wireless power transfer. However, due to intrinsic losses in the resistance of the inductor, the radiated EM field, and other factors, unless high transfer efficiencies are achieved, it is useless to attempt to transmit electrical energy wirelessly. Thus, the main, most important goal of a resonant coupled system is to maximize efficiency.

The aim of this experiment is to determine whether the efficiency of energy transfer in a resonant inductive system is correlated to the resonant frequency \( f_{\text{res}} \) used. A correlation would most likely imply an optimal frequency range for maximum energy transfer.

Hypothetically, if the entire system were modelled as an air-cored transformer, a higher frequency would lead to a higher voltage received. This does not imply a higher efficiency. Yet higher frequencies would result in less energy wasted by the coil. I believe this factor is more important, and hence I predict efficiency is positively correlated with frequency.
2. Experimental Design

2.1 Measuring Efficiency

The difficulty of this experiment lies in measuring the efficiency of the power transfer. For purely resistive components, power is the product of voltage and current, i.e. \( P = VI \). But because capacitors and inductors are reactive elements, phase shift is introduced between voltage and current. The formula \( P = VI \) must be generalized to \( P = V_{\text{RMS}} I_{\text{RMS}} \cos \phi \), where \( V_{\text{RMS}} \) is the root-mean-square of the voltage waveform, \( I_{\text{RMS}} \) is the root-mean-square of the current waveform, and \( \phi \) is the phase shift between the voltage and current waveforms. While voltage and current are simple to measure, phase shift is not. This was a major issue in experimental design.

Fortunately, however, phase shift is not present in the output of the AC source of the primary circuit. This is due to the following formula:

\[
\tan(\phi) = \frac{\omega L - 1}{\omega C}
\]

In this equation for phase shift in RLC circuits, \( \phi \) is the phase shift and \( \omega \) is the angular frequency of the AC waveform. At \( \omega = f_{\text{res}} \), \( \omega L = 1/\omega C \), and hence \( \phi = 0 \). The total power provided by the AC source is then simply \( P = V_{\text{RMS}} I_{\text{RMS}} \cos 0 = V_{\text{RMS}} I_{\text{RMS}} \).

For the secondary circuit, there is also a simple solution. Since voltage and current are in phase across a resistor, attaching a resistor (\( R_1 \)) of a precise, known value in series with the coil allows the measurement of power through \( P = V_{R_1}^2/R \), where \( V_{R_1} \) is the voltage drop across the resistor of resistance \( R \) (Figure 2-1). This gives the total power received by the secondary circuit.

![Figure 2-1: Schematic diagram of experimental system](image)

Thus, the efficiency of the entire system in Figure 2-1 is given by the equation

\[
\text{Efficiency} = \frac{\text{Power}_{\text{out}}}{\text{Power}_{\text{in}}} = \frac{V_{R_1}^2}{R} = \frac{V_{R_1}^2}{R_{\text{in}}V_{\text{src}} \times I_{\text{src}}}
\]

where \( V_{\text{src}} \) and \( I_{\text{src}} \) are the RMS voltage and RMS current of the AC source, respectively.

---

2.2 Apparatus Overview

Two LC circuits were constructed on small breadboards, and the two handmade coils were placed parallel to each other. While changing either L or C alters $f_{res}$, it is easier to change capacitors than make coils for every $f_{res}$ necessary; hence, the capacitors in both circuits were varied to achieve different resonant frequencies. A function generator drives the primary circuit, and test instruments measure current and voltage at appropriate places (Figure 2-2).

![Figure 2-2: Schematic diagram of experimental setup](image)

2.3 Materials List

Figure 2-3 is a list of all materials that were used in the experiment, including materials and tools necessary for the construction of the coils that were part of the LC circuits.

<table>
<thead>
<tr>
<th>Apparatus and Tools</th>
<th>Electrical components</th>
</tr>
</thead>
<tbody>
<tr>
<td>− Alligator clips and connectors</td>
<td>− Two solenoids (see section 2.3.1)</td>
</tr>
<tr>
<td>− 2 standard breadboards</td>
<td>− Various capacitors (see section 2.3.2)</td>
</tr>
<tr>
<td>− PVC tubing</td>
<td>− 10-ohm resistor</td>
</tr>
<tr>
<td>− 24 AWG magnet wire</td>
<td></td>
</tr>
<tr>
<td>− Oscilloscope</td>
<td></td>
</tr>
<tr>
<td>− Voltage probes</td>
<td></td>
</tr>
<tr>
<td>− Function generator</td>
<td></td>
</tr>
<tr>
<td>− Multimeter (with capacitance and AC current functions)</td>
<td></td>
</tr>
</tbody>
</table>

![Figure 2-3: List of materials used in experiment](image)
2.4 Descriptions of Certain Materials

2.4.1 PRIMARY AND SECONDARY COILS (L1, L2)

Each coil is an 80-turn air-cored solenoid made using 24 AWG magnet wire tightly wound around PVC tubing with an outer diameter of 107mm. Both have an inductance of 756 ± 2 µH, as measured with a DMM. The windings are taped into place.

2.4.2 CAPACITORS (C1, C2)

A range of ceramic capacitors were selected and used. Due to the 10% tolerances of the capacitors available, each pair of capacitors was picked and measured using a DMM to ensure that both capacitors in the pair had as similar values as possible. Values used are (±1%, as measured): 0.130nF, 0.970nF, 3.01nF, 4.30nF, 10.1nF, 14.4nF, 21.2nF, 33.7nF, 51.0nF, and 127nF.

2.4.3 BREADBOARDS

Two standard small-sized breadboards were used to connect components, to enable easy substitution of capacitors. For the purposes of this experiment, the capacitance, resistance, and any other effect of the breadboard traces were ignored because of their marginal effects on the final results.

2.4.4 INSTRUMENTS

A Circuit-Test SWF-7000 function generator provided the 50% duty cycle pure AC sine wave driving the primary circuit at 5.00 ± 0.01 V_{RMS}. While it can be tuned anywhere from 0.000 to 2.310 MHz, only the 20 to 600 kHz
range was used for this experiment.

A Tektronix TDS 1002B dual-channel oscilloscope was used to measure the output of the function generator, ensuring it is at 5.00 V\textsubscript{RMS}. It also measured the voltage drop across R1, a 10.0 ± 0.01 ohm resistor, to determine the total power in the secondary circuit. Two Tektronix P2220 voltage probes were used. Note that the oscilloscope is configured to measure the RMS and maximum voltage of both channels.

A Meterman 37XR digital multimeter measured the current output of the function generator. It was also used to measure the capacitors and inductance of the coils with its capacitance and inductance settings. The method it uses to calculate RMS readings is unknown. However, because all measurements should be pure sine waves, this is not a major issue, since even if it only calculates \( V_{\text{MAX}} / \sqrt{2} \) it will have an accurate RMS reading for a pure sine wave.

2.5 Procedure

All instruments and circuitry were set up as per Figure 2-2. The primary and secondary coils were taped together, and the distance between them was 4.4 ± 0.1cm. This distance is not relevant for the purposes of this experiment, as it is less than the diameters of the coils and hence all radiation is near field. More importantly, distance is not being varied as a variable in this experiment, and it is effectively controlled.

For each capacitance tested, the function generator was adjusted so the source waveform remains at 5.00 V\textsubscript{RMS} (as measured by channel A on the oscilloscope). The frequency knob of the function generator was adjusted until \( V_{R1} \) (as measured by channel B) was at a maximum and \( I_{\text{src}} \) (as measured by the DMM) was at a minimum. \( V_{R1} \) and \( I_{\text{src}} \) were then recorded. The capacitors on both sides were removed by hand after recording data and the next pair of capacitors was inserted into the breadboards. This procedure was repeated for all 10 capacitance values.

Because of body capacitance and other effects of direct contact with the apparatus, every reading was taken after a few seconds of replacing the capacitors, to allow the apparatus to settle.
down and the instruments to give values that did not fluctuate wildly. All readings were taken standing away from the apparatus, as even putting a hand near the coils would affect the oscilloscope readings.

Several trials were taken due to varying $I_{src}$ values, mostly likely due to changes in temperature and other environmental factors that possibly affect capacitance values and such in the circuitry as well as the measuring instruments.

Figure 2-10: Photograph of experimental setup
3. Data Analysis

3.1 Processed Raw Data

Before any analysis was performed on the collected data, it was necessary to convert the capacitance values to their equivalent $f_{\text{res}}$ values for easier analysis, as this experiment deals with the relationship between $f_{\text{res}}$ and efficiency, not capacitance and efficiency. This was done using the equation for resonant frequency, where $L$, as measured, is equal to 756 ± 2 µH:

$$f_{\text{res}} = \frac{\omega_0}{2 \pi} = \frac{1}{2 \pi \sqrt{LC}}$$

<table>
<thead>
<tr>
<th>$C$ (nF, ±1%)</th>
<th>0.130</th>
<th>0.970</th>
<th>3.01</th>
<th>4.30</th>
<th>10.1</th>
<th>14.4</th>
<th>21.2</th>
<th>33.7</th>
<th>51.0</th>
<th>127</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{\text{res}}$ (kHz, ±1%)</td>
<td>508</td>
<td>186</td>
<td>106</td>
<td>88.3</td>
<td>57.6</td>
<td>48.2</td>
<td>39.8</td>
<td>31.5</td>
<td>25.6</td>
<td>16.2</td>
</tr>
</tbody>
</table>

**Figure 3-1: Capacitances and their corresponding resonant frequencies**

However, note that while this equation provides the $f_{\text{res}}$ of an ideal LC system, the setup of this experiment was far from ideal, and hence the actual measured $f_{\text{res}}$ values probably differed slightly from those given by the equation. This is negligible, though, because it is unnecessary to know the exact $f_{\text{res}}$ to show a relationship between $f_{\text{res}}$ and efficiency, e.g. whether $f_{\text{res}}$ is 101kHz or 103kHz will not drastically affect whether a correlation is found. It is more important to have accurate values for efficiency than for the resonant frequencies. Hence in the data analysis, the ideal $f_{\text{res}}$ values given by the equation were used instead of the empirically determined $f_{\text{res}}$ values, which varied slightly across trials.

Upon measuring $V_{R1}$ and $I_{\text{src}}$, the formula for efficiency presented in section 2.1 was used to calculate the set of efficiency values for each trial (Figure 3-2). The original raw data is available in Appendix A.

$$\text{Efficiency} = \frac{V_{R1}^2}{R_1 V_{\text{src}} I_{\text{src}}} \times 100\%$$

Uncertainties for $f_{\text{res}}$ and efficiency values were derived by taking the maximum uncertainty present in any data point and using that uncertainty in the formula above. For example, the maximum uncertainty in $V_{R1}$ of trial 1 in the raw data is 0.685%, from 146 ± 1mV.
<table>
<thead>
<tr>
<th>$f_{\text{res}}$ (kHz, ±1%)</th>
<th>Efficiency (1) (%) ±3%</th>
<th>Efficiency (2) (%) ±3%</th>
<th>Efficiency (3) (%) ±3%</th>
<th>Efficiency (4) (%) ±3%</th>
</tr>
</thead>
<tbody>
<tr>
<td>16.2</td>
<td>5.81</td>
<td>6.03</td>
<td>6.74</td>
<td>8.47</td>
</tr>
<tr>
<td>25.6</td>
<td>11.2</td>
<td>18.2</td>
<td>15.7</td>
<td>17.0</td>
</tr>
<tr>
<td>31.5</td>
<td>30.5</td>
<td>19.8</td>
<td>34.7</td>
<td>27.5</td>
</tr>
<tr>
<td>39.8</td>
<td>30.4</td>
<td>30.3</td>
<td>32.7</td>
<td>25.8</td>
</tr>
<tr>
<td>48.2</td>
<td>22.9</td>
<td>29.2</td>
<td>23.0</td>
<td>33.8</td>
</tr>
<tr>
<td>57.6</td>
<td>25.4</td>
<td>24.3</td>
<td>26.9</td>
<td>21.8</td>
</tr>
<tr>
<td>88.3</td>
<td>57.2</td>
<td>43.7</td>
<td>49.3</td>
<td>53.7</td>
</tr>
<tr>
<td>106</td>
<td>36.8</td>
<td>41.8</td>
<td>23.4</td>
<td>43.0</td>
</tr>
<tr>
<td>186</td>
<td>28.0</td>
<td>28.7</td>
<td>33.8</td>
<td>33.0</td>
</tr>
<tr>
<td>508</td>
<td>42.2</td>
<td>41.2</td>
<td>49.0</td>
<td>46.2</td>
</tr>
</tbody>
</table>

**Figure 3-2: Processed raw data showing resonant frequencies and efficiencies**

Looking at the table, it appears that as resonant frequency is increased, efficiency increases. A graph of the relationship between resonant frequency and efficiency is shown below.

**Efficiency of transfer at different resonant frequencies**

![Graph of processed raw data](image)

**Figure 3-3: Graph of processed raw data**
3.2 Measuring correlation and significance

A visual inspection of the graph of the processed raw data (Figure 3-3) suggests that there is a positive correlation between efficiency and resonant frequency. To examine this possible correlation in more detail, the data from all four trials was combined into one set of values. Because there were no clear outliers within each set of four efficiencies, taking the average of the four efficiencies for each point provided a suitable set of data for analysis. The resulting data set is graphed in Figure 3-4 below.

![Average efficiencies of all trials](image)

Figure 3-4: Average efficiency of each frequency tested

A statistical test was performed on the data to determine the strength of the correlation. Though Pearson’s product moment correlation coefficient is a popular choice for measuring the correlation between two variables, it only tests for linear dependence.² From Figure 3-4, it is obvious that the relationship between efficiency and resonant frequency is not linear: a linear relationship would be an exponential curve on a logarithmic scale, and the data points do not

---

seem to fall on such a curve. Hence, Spearman’s rank correlation coefficient was chosen as an appropriate statistical test of correlation, as it is not limited to testing purely linear dependences.

<table>
<thead>
<tr>
<th>$F_{\text{res}} (X_i)$</th>
<th>Eff. (Y$_i$)</th>
<th>Rank (X$_i$)</th>
<th>Rank (Y$_i$)</th>
<th>$d_i^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>16.2</td>
<td>6.76</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>25.6</td>
<td>15.5</td>
<td>2</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>31.5</td>
<td>28.1</td>
<td>3</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>39.8</td>
<td>29.8</td>
<td>4</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>48.2</td>
<td>27.2</td>
<td>5</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>57.6</td>
<td>24.6</td>
<td>6</td>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>88.3</td>
<td>51.0</td>
<td>7</td>
<td>10</td>
<td>9</td>
</tr>
<tr>
<td>106</td>
<td>36.3</td>
<td>8</td>
<td>8</td>
<td>0</td>
</tr>
<tr>
<td>186</td>
<td>30.9</td>
<td>9</td>
<td>7</td>
<td>4</td>
</tr>
<tr>
<td>508</td>
<td>44.7</td>
<td>10</td>
<td>9</td>
<td>1</td>
</tr>
</tbody>
</table>

**Figure 3-5: Calculation of Spearman’s correlation coefficient**

Upon ranking the data and calculating the correlation, the following value was obtained:

$$\rho = 1 - \frac{6 \sum d_i^2}{n^3 - n} = 1 - \frac{192}{990} \approx 0.806$$

This indicated a strong positive correlation between efficiency and resonant frequency. For a two-tailed test, the critical value at a 5% significance level for 8 degrees of freedom is 0.738, while the value at the 2% significance level is 0.833. Hence, there is a strong positive correlation with more than 95% certainty.

However, this does not provide any indication of the nature of the correlation. From Figure 3-4, it appears that the best-fit curve would be a straight line, which would mean a logarithmic relationship between efficiency and resonant frequency due to the logarithmic scale of the x-axis. It was unclear, though, whether a logarithmic or other type of best fit curve would be suitable in representing the trend of the data. An easy way of determining this is to visually examine different types of best-fit curves.

---

3.3 Determination of Relationship

**Power curve of best fit**

\[ y = 4.61x^{0.416} \]

\[ R^2 = 0.539 \]

**Logarithmic curve of best fit**

\[ y = 9.57\ln(x) - 10.4 \]

\[ R^2 = 0.576 \]

*Figure 3-6: Curves of best fit for efficiency-frequency graph*
The two graphs in Figure 3-6, produced by Microsoft Excel using the “power” and “logarithmic” trend line functions, demonstrate that while efficiency increases quickly with lower resonant frequencies, the increase in efficiency slows down as higher and higher resonant frequencies are used. Both graphs have moderate R\(^2\) values because of this – both the \(x^{0.416}\) and \(\ln(x)\) functions exhibit this behaviour. Upon visual inspection, however, the logarithmic curve appears to fit better than the power curve, which reaches excessively high values of efficiency at higher resonant frequencies.

In contrast, Figure 3-7 is a linear curve of best fit. Evidently, it does not model the data points as closely as a power or logarithmic curve.

![Linear curve of best fit](image)

**Figure 3-7: Linear curve of best fit for efficiency-frequency graph**

Hence, the conclusion reached regarding the data was that the relationship between efficiency and resonant frequency is a logarithmic one, or at least pseudo-logarithmic like the power function with an exponential value between 0 and 1. This is supported by visual inspection of a straight line being the best-fit curve when the data is graphed on a logarithmic scale.
4. Discussion

4.1 Data and Results

A logarithmic relationship between efficiency and resonant frequency makes sense, given the following points. Firstly, at the extremely low frequencies, efficiency is very close 0, because there is little or no change in magnetic flux on the primary coil, and hence little or no energy is transferred at all, with the side effect of short-circuiting the function generator. Secondly, there is an upper limit on efficiency, as efficiency cannot exceed 100%, and ideal components do not exist. Finally, as long as there is a change in the magnetic flux, i.e. frequency is non-zero, there will be a certain amount of energy transferred. Therefore, an efficiency-frequency graph should begin close to the origin, with a fast rate of increase in efficiency as resonant frequency is increased, which gradually slows down as higher frequencies are reached. A logarithmic function seems to model this behaviour almost perfectly.

\[ \text{Figure 4-1: A generic logarithmic function} \]

However, there is one issue with a logarithmic relationship: \( \log_a(x) \) has no upper bound. Eventually, for large values of \( x \), or for sufficiently high resonant frequencies, the predicted efficiency will be over 100%, which does not make sense. For the logarithmic curve of best fit in Figure 3-6, it can be shown that for any frequency larger than 102MHz, efficiency is larger than 100%. Hence, a function with a horizontal asymptote at infinity is necessary, most likely something of the form \( 1 - x^a \). It is pointless to test this guess, though, with the limited points of data available and the already vague fit with the power and logarithmic functions.
4.2 Sources of Error and Improvements

There were many issues with experimental design. First and foremost was the unavailability of capacitors with similar characteristics. As the photo of the some of the capacitors (Figure 2-5) shows, some capacitors appear to be physically different from others. Due to the unknown specifications for the different capacitors, each capacitor was selected based on its capacitance only, as measured with the DMM. This means that some of the capacitors may have different electrical characteristics, and hence may behave differently in the parallel LC circuit used in this experiment. The 4.30nF capacitor appeared to produce higher efficiencies than the other capacitors, for example, even though the corresponding frequency, 88.3 kHz, is not significant in any way. Connecting the 3.01nF and 0.930nF capacitors in parallel to replace the 4.30nF capacitor gave an efficiency of 39.7%, which is in direct contrast with the 51.0% efficiency obtained from the 4.30nF capacitor. Hence, the peak in efficiency at 88.3 kHz that appears on the data and graphs can be attributed to varying electrical characteristics of the capacitors instead of possible strange effects causing efficiency increase in the 80-90 kHz frequency range.

Unfortunately, this issue is difficult to overcome. Since the ideal capacitor does not exist, there are many different varying electrical characteristics of the capacitors. Capacitances are known to change as the age of the capacitor increases, or as the surrounding temperature changes. Even though capacitance changes may be negligible, other changes in electrical characteristics may have resulted in abnormally high or low efficiencies. These changes are hard to prevent and difficult to quantify.

A second major issue arose during data collection. Due to the sensitivity of the instruments used, fluctuations of values being measured were present at any moment of the experiment. The function generator, for example, had an odd tendency to slowly drop in frequency after the initial adjustment of the frequency knob. This would cause the readings on the oscilloscope to fluctuate and eventually begin dropping as well. The oscilloscope’s RMS calculations accentuated this issue even more, as a slightly larger maximum voltage on one peak of the AC sinusoid would cause the RMS voltages to start fluctuating. Finally, measurements in the millivolt and milliamp ranges did not help to increase accuracy.

---

However, this issue was easier to resolve. While the fluctuations were moderately large and unpredictable initially after setting the correct frequency and components, they diminished after around ten seconds. This meant that more accurate readings could be taken after letting the apparatus settle down for half a minute or so, after which the function generator would stabilize. Repeating the experiment helps increase the accuracy as well, though it was very hard, if not impossible, to eliminate the noise inherent in the waveforms due to the sensitivity of the instruments.

Finally, though this is not technically a source of error, the apparatus and components were not designed to optimize efficiency in any way. Coils with higher Q-factors in a strongly coupled regime have been shown to exhibit higher efficiencies than other configurations. This was not considered at all during the construction of the apparatus, and hence the behaviour of the system in this experiment may differ from that of a strongly coupled resonant system, which may render the result that higher frequencies have higher efficiency inaccurate. However, this is unlikely, because though high efficiencies were not achieved, it was noted that removing the capacitor on the secondary circuit causes no power to be received on the secondary, which means that, at the very least, there is resonance, and hence the behaviour of the system should be similar, if not identical to, a strongly coupled system.

One major improvement that could have been made is to either construct a tapped coil to alter inductance, or to use a variable capacitor to alter capacitance. This would effectively resolve the issue of different capacitors having different electrical characteristics, because the same component would be in use throughout the experiment, only with different values. It is possible that doing so will eliminate the abnormally high or low efficiencies associated with certain frequencies, which have been shown to be caused by the different capacitors. Doing so would also narrow the range of the uncertainties present in the measurements.

Another possible improvement is to replace the function generator with a sine wave oscillator, e.g. a Colpitts oscillator. This would enable much easier measurements of input power, though the efficiency of the oscillator chosen would have to be taken into account as well, potentially making things more complicated.

---

5. Conclusion

The purpose of this experiment was to determine the relationship between the efficiency of a resonant inductive coupled system and its resonant frequency. Within the scope of this experiment, i.e. at moderately high frequencies from 20 to 600kHz, there appears to be a reasonable relationship. As the resonant frequency of the system increases, the efficiency increases, though at a slower and slower rate for higher resonant frequencies. An exact equation modelling the relationship could not be found, however it was determined that the equation must have a horizontal asymptote, i.e. it has a certain upper limit as resonant frequency approaches infinity. This supports my hypothesis of a positive correlation between frequency and efficiency. A regression also determined that the equation behaves pseudo-logarithmically, exhibiting a fast rate of increase for low frequencies and a slow rate of increase for higher frequencies.

This has several implications. First, and the most obvious, is that higher frequencies should be used in designing wireless power transfer systems using resonant inductive coupling. With the appropriate design and set up, this would maximize the efficiency and lower the percentage of wasted, dissipated energy. However, the second implication is that it is pointless to attempt to use the highest possible frequency with such a system, because past a certain point, the increase in efficiency with the increase of frequency is almost negligible, and because oscillators of higher frequencies tend to waste more power in generating waveforms, it is actually more inefficient to attempt to use frequencies higher than a certain point. Where this point is, though, is outside the scope of this experiment. Finally, though not officially part of the experiment, several capacitors exhibited irregular characteristics, in that they either caused an abnormally high or abnormally low efficiency. This may imply that certain electrical characteristics are desirable in the components of a resonant system, which may be of interest to future studies.

Further research on this topic should involve determining the efficiencies for higher frequencies than 600 kHz, both to attempt to verify the results of this experiment and also to determine, quantitatively, the value of the upper limit on efficiency and how that value is affected. Knowing how to increase the upper limit would be greatly beneficial to creating wireless power transfer systems at higher efficiencies.
### Appendix A: Raw Data

<table>
<thead>
<tr>
<th>Capacitance (nF, ±1%)</th>
<th>( I_{src} ) (mA, ±1%)</th>
<th>( V_{R1} ) (mV, ±1mV)</th>
<th>Capacitance (nF, ±1%)</th>
<th>( I_{src} ) (mA, ±1%)</th>
<th>( V_{R1} ) (mV, ±1mV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>127</td>
<td>17.9</td>
<td>228</td>
<td>16.2</td>
<td>262</td>
<td></td>
</tr>
<tr>
<td>51.0</td>
<td>11.2</td>
<td>250</td>
<td>10.6</td>
<td>300</td>
<td></td>
</tr>
<tr>
<td>33.7</td>
<td>7.57</td>
<td>340</td>
<td>5.91</td>
<td>285</td>
<td></td>
</tr>
<tr>
<td>21.2</td>
<td>6.32</td>
<td>310</td>
<td>4.46</td>
<td>240</td>
<td></td>
</tr>
<tr>
<td>14.4</td>
<td>4.62</td>
<td>230</td>
<td>4.31</td>
<td>270</td>
<td></td>
</tr>
<tr>
<td>10.1</td>
<td>4.53</td>
<td>240</td>
<td>3.14</td>
<td>185</td>
<td></td>
</tr>
<tr>
<td>4.30</td>
<td>3.58</td>
<td>320</td>
<td>2.87</td>
<td>281</td>
<td></td>
</tr>
<tr>
<td>3.01</td>
<td>2.44</td>
<td>212</td>
<td>1.92</td>
<td>241</td>
<td></td>
</tr>
<tr>
<td>0.970</td>
<td>1.36</td>
<td>138</td>
<td>1.26</td>
<td>151</td>
<td></td>
</tr>
<tr>
<td>0.130</td>
<td>1.01</td>
<td>146</td>
<td>0.930</td>
<td>170</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Capacitance (nF, ±1%)</th>
<th>( I_{src} ) (mA, ±1%)</th>
<th>( V_{R1} ) (mV, ±1mV)</th>
<th>Capacitance (nF, ±1%)</th>
<th>( I_{src} ) (mA, ±1%)</th>
<th>( V_{R1} ) (mV, ±1mV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>127</td>
<td>15.7</td>
<td>230</td>
<td>16.2</td>
<td>262</td>
<td></td>
</tr>
<tr>
<td>51.0</td>
<td>9.70</td>
<td>276</td>
<td>10.6</td>
<td>300</td>
<td></td>
</tr>
<tr>
<td>33.7</td>
<td>6.66</td>
<td>340</td>
<td>5.91</td>
<td>285</td>
<td></td>
</tr>
<tr>
<td>21.2</td>
<td>4.90</td>
<td>283</td>
<td>4.46</td>
<td>240</td>
<td></td>
</tr>
<tr>
<td>14.4</td>
<td>3.52</td>
<td>201</td>
<td>4.31</td>
<td>270</td>
<td></td>
</tr>
<tr>
<td>10.1</td>
<td>3.37</td>
<td>213</td>
<td>3.14</td>
<td>185</td>
<td></td>
</tr>
<tr>
<td>4.30</td>
<td>2.87</td>
<td>266</td>
<td>2.94</td>
<td>281</td>
<td></td>
</tr>
<tr>
<td>3.01</td>
<td>1.92</td>
<td>150</td>
<td>2.70</td>
<td>241</td>
<td></td>
</tr>
<tr>
<td>0.970</td>
<td>1.26</td>
<td>146</td>
<td>1.38</td>
<td>151</td>
<td></td>
</tr>
<tr>
<td>0.130</td>
<td>0.930</td>
<td>151</td>
<td>1.25</td>
<td>170</td>
<td></td>
</tr>
</tbody>
</table>
Works Cited


Works Consulted


